Extreme value modelling of clinical trial safety data

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Overview

- Specification of the problem
- Introduction to extreme value modelling
- Univariate analysis
- Multivariate analysis
Safety data from clinical trials

- Second most common reason for late phase failure is safety
- Most common reason for market withdrawal is safety
- Why?
  - Trials designed to address a well-specified efficacy question
  - Most of the data relate to safety
    - Adverse events, lab data, vital signs, etc.
  - A mixture of binomial, time-to-event, and continuous but highly non-Gaussian data
  - Impossible to avoid interest in data driven questions
    - Leads to concern over lack of pre-specification, multiplicity, power
Expected values can’t help

- Most statistical methods are designed to predict **expected values**
  - Mean, median, regression models
- With safety, it’s usually the **unexpected values** that are of interest
  - Unusually big or small values in labs, vital signs
  - There is no reason to suppose that the behaviour of the data near the centre of the distribution is informative about values in the tails
  - To characterize the centre and assume it tells us something about the safety of the drug can be misleading and dangerous

- So, **the usual statistical methods can’t help!**
Extreme values of ALT

- The rest of the presentation focuses on extremes of liver related lab variables
  - Large values of ALT suggest potential liver injury
  - Potential for liver injury has been the most frequent reason for safety related withdrawal of drugs from the market [1]
    - ticrynafen, benoxaprofen, bromfenac, troglitazone, nefazodone, ximelagatran...
Extreme values of ALT (2)

According to published guidance (CTC [3]):

<table>
<thead>
<tr>
<th>Grade</th>
<th>Severity</th>
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<tbody>
<tr>
<td>1</td>
<td>Mild</td>
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<tr>
<td>2</td>
<td>Moderate</td>
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<tr>
<td>3</td>
<td>Severe</td>
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<tr>
<td>4</td>
<td>Life threatening</td>
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</table>

ULN can be thought of as the units of measurement for ALT.
Example: troglitazone

Troglitazone (for treatment of diabetes). FDA review states

Mean [ALT] levels fell in patients receiving troglitazone in phase 3 trials... It was also stated that 2.2% of patients in phase 3 trials had an [ALT] level exceeding $3 \times ULN$... What was not appreciated by [FDA] was that many of the patients classified as $ALT > 3 \times ULN$ actually had ALT values that were VERY much higher than $3 \times ULN$... 23 patients had treatment-emergent ALT values over $3 \times ULN$... In 14 of these 23 patients, the ALT value exceeded $8 \times ULN$... and in 5/23 patients the ALT value exceeded $30 \times ULN$.

The drug was withdrawn from the market after reports of liver failure and death
ALT isn’t the full story

Hy’s Law

So far as I (Harry) understand what I’ve read...

- If ALT or AST is ‘high’, then
- If total bilirubin (TBL) is ‘high’, then
  - If alkaline phosphatase (ALP) is normal
    - Conclude drug induced liver injury
  - If ALP is ‘high’
    - Conclude underlying liver disease

Which logically implies that if we only develop drugs that cause ALP elevation, we cannot cause liver injury

- ... which seems problematic
According to the Health Canada guideline [2]

- ‘It has been noted that $ALP > 2 \times ULN$ occurs in one third of potential Hy’s Law cases and can be associated with subsequent liver failure’


So the interpretation of data is

- Multivariate
- Disputed
Extreme value modelling

- A mature branch of statistics
  - Key publication by R. A. Fisher and L. H. C. Tippett, 1928
  - First full length textbook by E. Gumbel, 1958

- Used in many areas of application:
  - Meteorology, insurance, finance, geology, metallurgy, materials science, network traffic, ...
We wish to describe the tails of a distribution

- **extrapolation** – at your own risk!!
- parametric model needed
- calibrate using **data from tails**

... but something **more reassuring** is needed.
Asymptotically motivated statistical models

Recall… **Central Limit Theorem**

- mean $\overline{X}_n$ of IID observations $X_1, \ldots, X_n$ mean $\mu$, finite variance $\sigma$
- $\overline{X}_n \rightarrow \mu$ in probability
- normalised $\overline{X}_n$

\[
\frac{\overline{X}_n - \mu}{\sigma / \sqrt{n}} \rightarrow \text{Normal}(0, 1) \text{ as } n \rightarrow \infty,
\]
Asymptotic argument for choice of statistical model

Ingredients:

- IID data (distribution unknown)
- operation induces stability (for CLT this is SUMMING)
- normalisation to avoid degeneracy
- limit distribution in parametric family (for CLT this is NORMAL)

Limit argument used for finite samples when $n$ is "sufficiently large".
Our interest is in the tails of the distribution

Do the same as in the CLT but use **Maximum** instead of **Mean**

This gives us the **Extremal Types Theorem**

- $\max M_n$ of IID observations $X_1, \ldots, X_n$
- $M_n \rightarrow$ upper end point of distribution in probability
- normalised $M_n$ – if limit exists then

$$\frac{M_n - a_n}{b_n} \rightarrow \text{GEV}(\xi) \text{ as } n \rightarrow \infty,$$

GEV is the **Generalised Extreme Value Distribution**.
Model for threshold excesses

- use the **GEV for maxima**
- derive model for tails of distribution $F$ of $X_1, \ldots, X_n$
Generalised Pareto distribution

If assumptions behind GEV hold then:

- there exists **threshold** $u$ excesses of which are
  \[ Y_u = (X - u)_+ . \]

- then $Y_u | Y_u > 0 \sim \text{GPD}(\sigma_u, \xi)$
  \[
  \Pr(Y_u < y | Y_u > 0) = 1 - [1 + \xi y / \sigma_u]_+^{-1/\xi},
  \]
The generalized Pareto distribution

Two parameters:
- $\sigma_u$: the scale parameter
- $\xi$: the shape parameter
  - $\xi < 0$: short tailed distribution (finite upper end point)
  - $\xi = 0$: exponential distribution
  - $\xi \geq 0$: heavy tailed distribution

- no straightforward physical meaning
  - Results often presented using
    - Predicted probabilities of exceeding certain thresholds
    - Predicted extreme quantiles of the distribution – return levels
Univariate results

Data from a single clinical trial
- Patients randomized to one of four doses of drug
  - Doses coded A (lowest), B, C, D (highest)
- Approximately 160 patients per dose
- A baseline assessment of ALT, AST, TBL, ALP
- And a single on-treatment assessment

The highest dose of the drug has been associated with liver injury in the literature

First, let’s look at ALT alone
Shiftplots of ALT

Baseline ALT (U/L) vs. On-treatment ALT (U/L) for different doses:
- Dose A
- Dose B
- Dose C
- Dose D
Boxplots of residuals after removing baseline effect

Scaled residuals

A  B  C  D
GPD models for ALT

Consider models with term for dose in
- $\phi = \log \sigma$
- $\xi$
- $\phi$ and $\xi$

Choose between models using Akaiki’s Information Criterion
- Model with linear term for dose in $\xi$ is preferred
- $\hat{\xi} < 0$ for doses A and B, $\hat{\xi} > 0$ for C and D
Predicted 1000-patient return levels
Predicted probabilities of threshold exceedance

- $P(ALT > ULN)$
- $P(ALT > 3xULN)$
- $P(ALT > 10xULN)$
- $P(ALT > 20xULN)$

P( > RL)
What is a multivariate extreme value?

- componentwise maxima?
- points that are large in
  - all components?
  - any component?
  - a given component?
Consider multivariate response \((X, Y)\).

- work with standardised margins
- look at

\[
\lim_{u \to \infty} \Pr(Y < y \mid X > u)
\]

limiting form of conditional distribution as threshold \(u \to \infty\).

- if \(X\) and \(Y\) are at all dependent then for any fixed \(y\) this limit will be degenerate
- need to center and scale \(Y\) to track its growth with \(X\)
Standardised Laplace margins are convenient (the maths works!)

Then the appropriate centering and scaling is:

\[
Y - ax
\]

\[
\frac{x^b}{x^b}
\]

for large values \( x \) of conditioning variable \( X \).

There are two model parameters:

- \( a \) describes change of location with threshold
- \( b \) describes change in scale with threshold
Regression interpretation

Defining

\[ Z = \frac{Y - ax}{x^b} \]

we can think of this as a regression model:

\[ Y = ax + x^b Z \]

\( Z \) are residuals, assumed independent of explanatory variable \( x \).
Detailed model specification

**Marginal** model for each variable

- **GPD tail** above threshold
- **empirical** distribution below threshold
- used to **transform margins** to standard (Laplace) margins

**Dependence** model

- threshold model – **fit to data for which** $X > v$ for high $v$
- **parameters** $a$ and $b$ capture dependence of $Y$ on $X$
- residuals $Z$ assumed **independent** of $X$ above threshold
Sample based extrapolation

Sample from distribution of $(X, Y)$ given $X > t$ for $t > v$

1. simulate value $x$ from tail of $X | X > t$
2. sample from observed model residuals $Z$
3. construct

   \[ Y = ax + x^b Z \]

4. $(X, Y)$ is an importance sample from the distribution of

   \[ (X, Y) | X > t \]
From the clinical trial described previously, look at **TBL and ALP as well as ALT**

- Ignore AST to simplify output

Remember:

- The highest dose has been associated with liver injury in the literature
- Traditional wisdom has it that liver injury results in **high ALT and TBL, but not high ALP**

Here we’ll focus on **proportional change from baseline**
Observed TBL vs ALT

Proportional change in TBL vs Proportional change in ALT

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Proportional change in ALT

Proportional change in TBL

Graph showing the relationship between proportional changes in TBL and ALT for different scenarios A, B, C, and D.
Observed ALP vs ALT
Predicted and observed TBL vs ALT
Predicted and observed ALP vs ALT
Some comments

▶ From experience, the relationship between ALT and ALP in this example is *not* typical

▶ The implication is that *some drugs associated with liver injury cause ALP to rise*

▶ Possibly, the lack of relationship between ALT and TBL is due to the short duration of treatment
Experience to date suggests

- Extreme value modelling really can predict toxicities from early phase data
- Approximately 50% of the time, outliers that appear worrying turn out to be consistent with no treatment effect
  - “Proceed with caution” rather than “clean bill of health”
- Focus here has been on liver, but we’ve also analysed
  - Creatinine, LVEF, neutrophils, blood pressure
U.S. Food and Drugs Administration.
2008.

Health Canada.
*Guidance Document: Pre-market Evaluation of Hepatotoxicity in Health Products.*
2012.

DCTD, NCI, NIH, and DHHS.
*Cancer Therapy Evaluation Program, Common Terminology Criteria for Adverse Events, Version 3.0.*
2003.

N. Kaplowitz.
*Does elevated alkaline phosphatase exclude Hy’s Law? No.*
2010.