

**PSI 2024 Conference  
Career Young Session**

**June 19<sup>th</sup>, 2024**

clement.daniel@servier.com

**Controlled multiple imputation in  
time-to-event data using tipping  
point analysis**

Daniel C., Rincourt S., Delaporte F. and Skanji D.

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# Overview

- I. Missing data in survival analysis
- II. Tipping Point Analysis (TPA)
- III. Simulation and results
- IV. Conclusion

# Missing data in survival analysis

## Context

In survival analysis context, when the event can't be observed we talk about **censoring**.

The **censoring reason** can be:

**Administrative**  
(end of study, cut-off)

**Non-administrative** (lost to follow-up, treatment discontinuation).

**Administrative** censoring can be considered as **ignorable** and **non-informative**

➤ **Informative censoring assumption (CAR).**

**Non-administrative** censoring is more likely to be **related to study treatment** and to be considered as **non-ignorable (informative censoring, CNAR)**.

**Informative censoring can lead to biased estimates.**

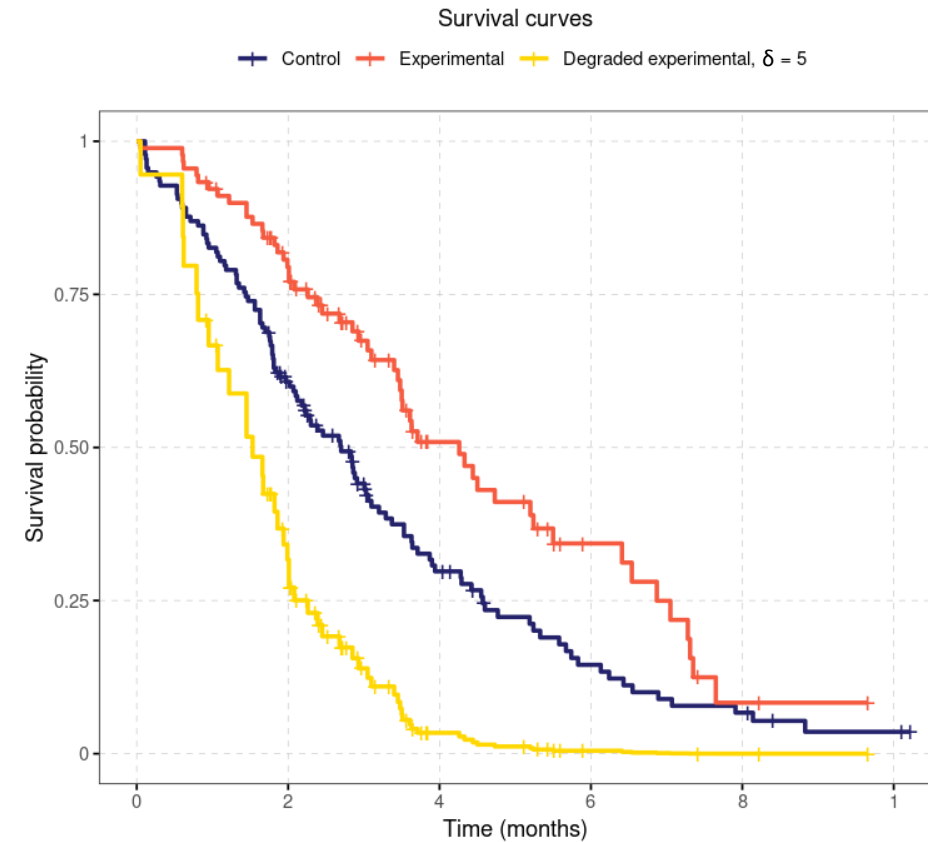
# Missing data in survival analysis

Methods for missing data: Tipping point analysis

Sensitivity analyses can be performed to evaluate the **robustness of the endpoint results** to deviations from the **ignorable censoring assumption (CAR)**.

**Tipping point analysis (TPA)** is a sensitivity analysis that is increasingly requested by **health authorities**.

- TPA is based on:
  - Survival model (e.g. Cox, Kaplan Meier) imputation,
  - Controlled multiple imputation.
- TPA consists of:
  - Incrementally** penalizing (by  $\delta$ ) the imputed event times in the experimental arm until the result between the 2 groups is **no longer statistically significant** (tipping point)



# The Tipping Point - interpretation

Tipping Point is defined as the lowest  $\delta$  value for which the result between the 2 arms is no longer statistically significant

- Example: A **Tipping Point** equal to **3** would mean that:  
in order to **switch our results** to non-significant,  
the **hazard** following discontinuation of informatively censored participants from the experimental arm would need to be **3 times larger** than the hazard of similar participants remaining in the study.

The **greater** the Tipping Point, the more **robust** the results are to **deviation** from the **ignorable censoring assumption** (CAR).

# Objectives

➤ Stress-test the results under the non-ignorable censoring assumption (CNAR)

➤ Study the accuracy of the Tipping Point Analysis methods

➤ Identify the parameters of a clinical trial that drive the value of  $\delta$

# Method: Tipping point analysis in survival analysis

For a participant  $i$  discontinued at time  $c_i$ ,

- let  $h_1(t)$  be his hazard at any given time point  $t$  following discontinuation,
- let  $h_2(t)$  be his hazard at the same time  $t$  if he/she had continued the study.

$$h_1(t) = \delta * h_2(t)$$

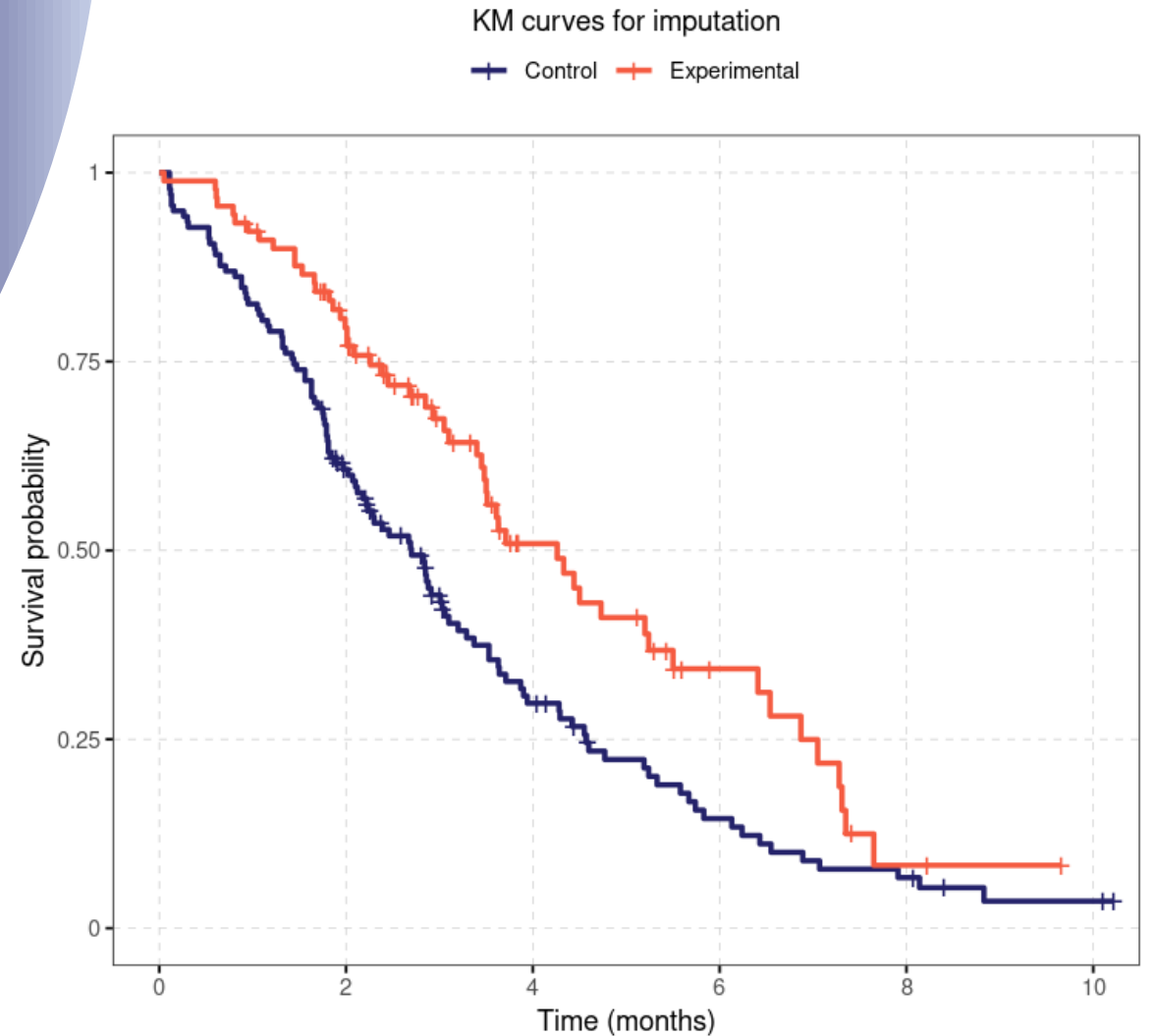
- **$\delta = 1$ , same hazard** following discontinuation as if he had remained in the study,
- **$\delta > 1$ , greater hazard** following discontinuation than the one he would've had

**$\delta$  penalty** is only applied to:

- **Informatively** censored participants
- Participants from the **experimental** arm

# Algorithm

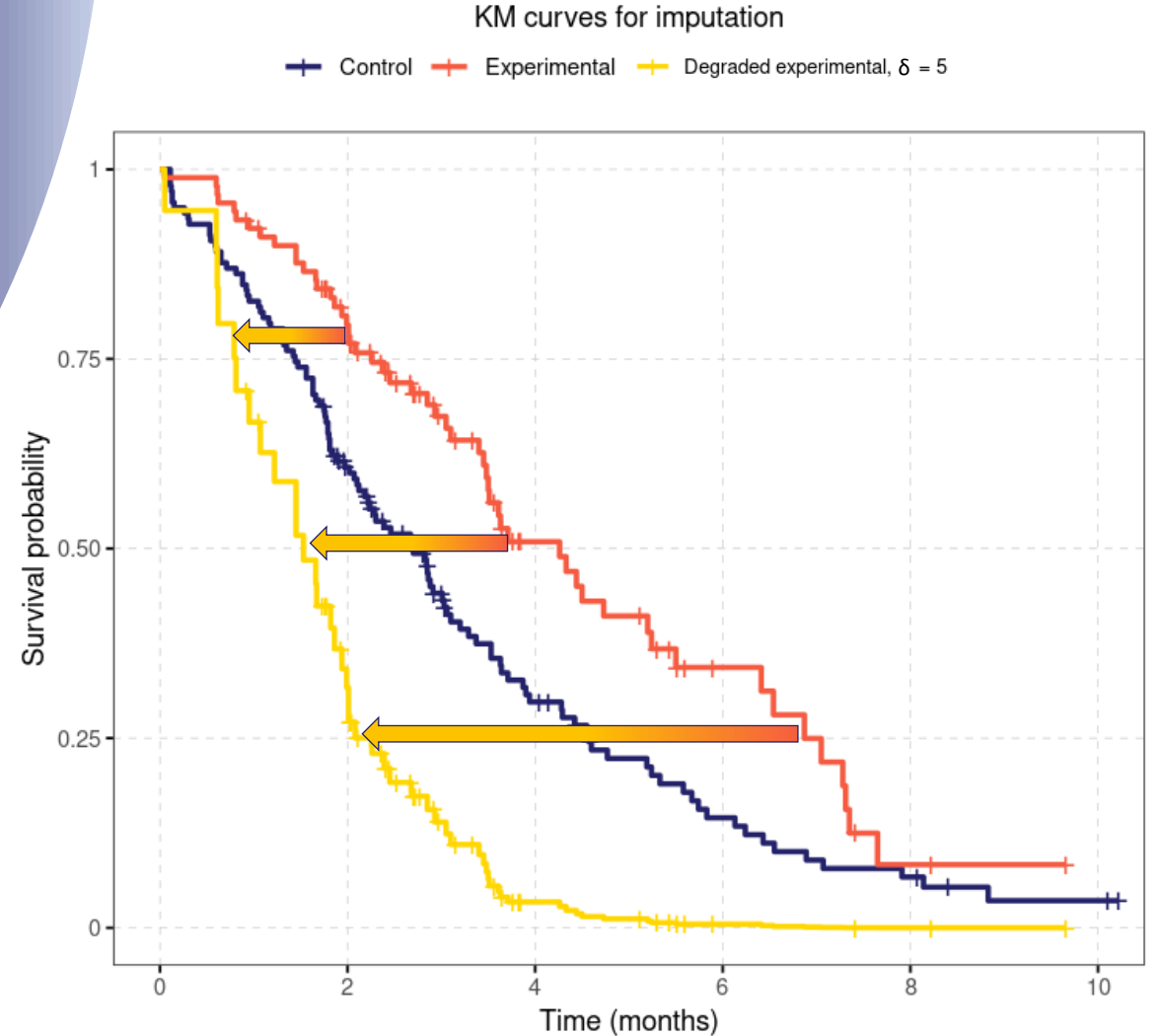
1. Evaluate the survival function  $S(t)^\delta$
2. For a participant  $i$  censored at time  $c_i$ , let  $p_i = \hat{S}(c_i|x_i, \hat{\beta})^\delta$
3. Draw  $u_i \sim U(0, p_i)$
4. Impute the event time  $t_i^*$  as the solution of  $u_i = \hat{S}(t|x_i, \hat{\beta})^\delta$
5. Analyze the new dataset with imputed event times





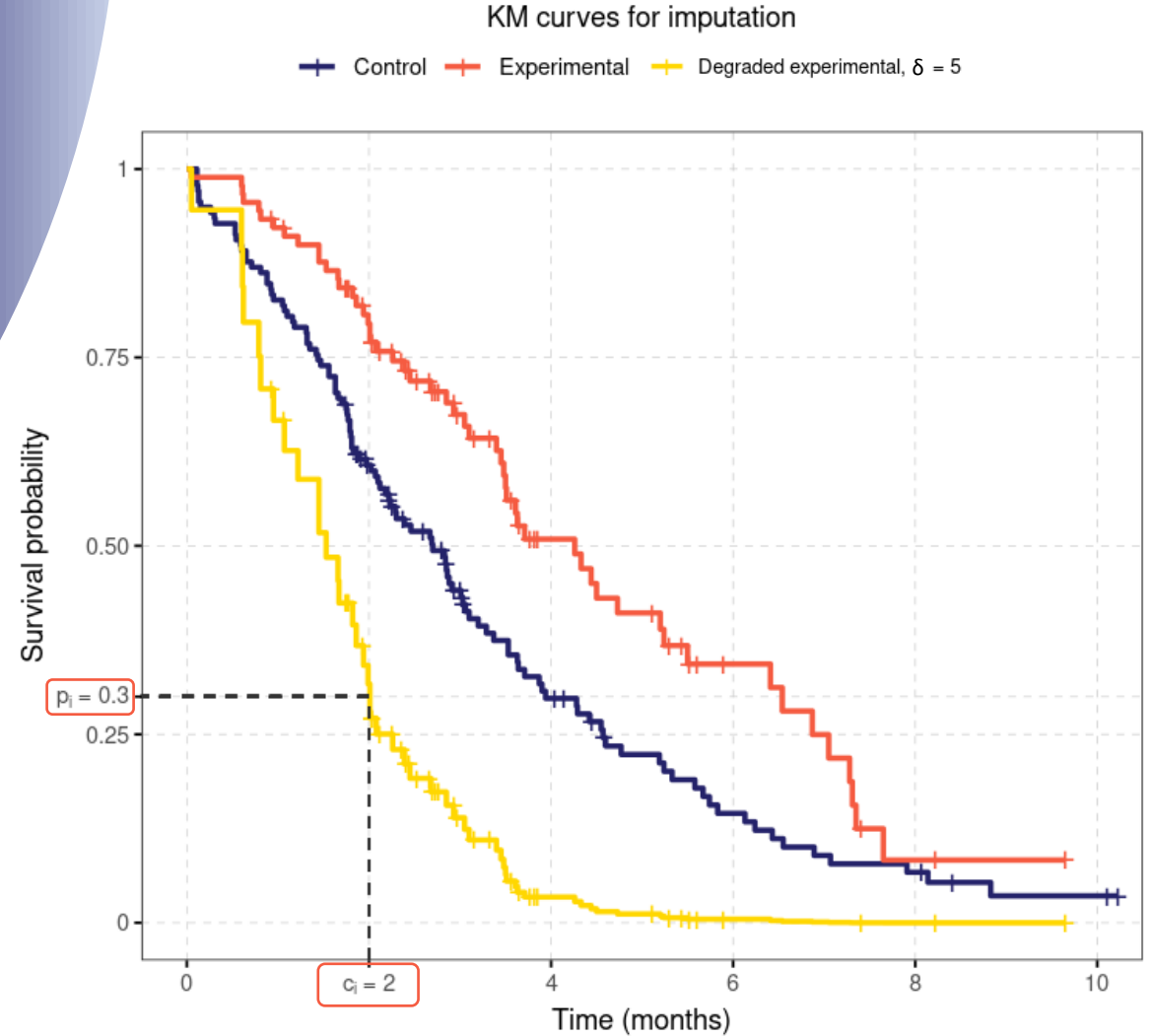
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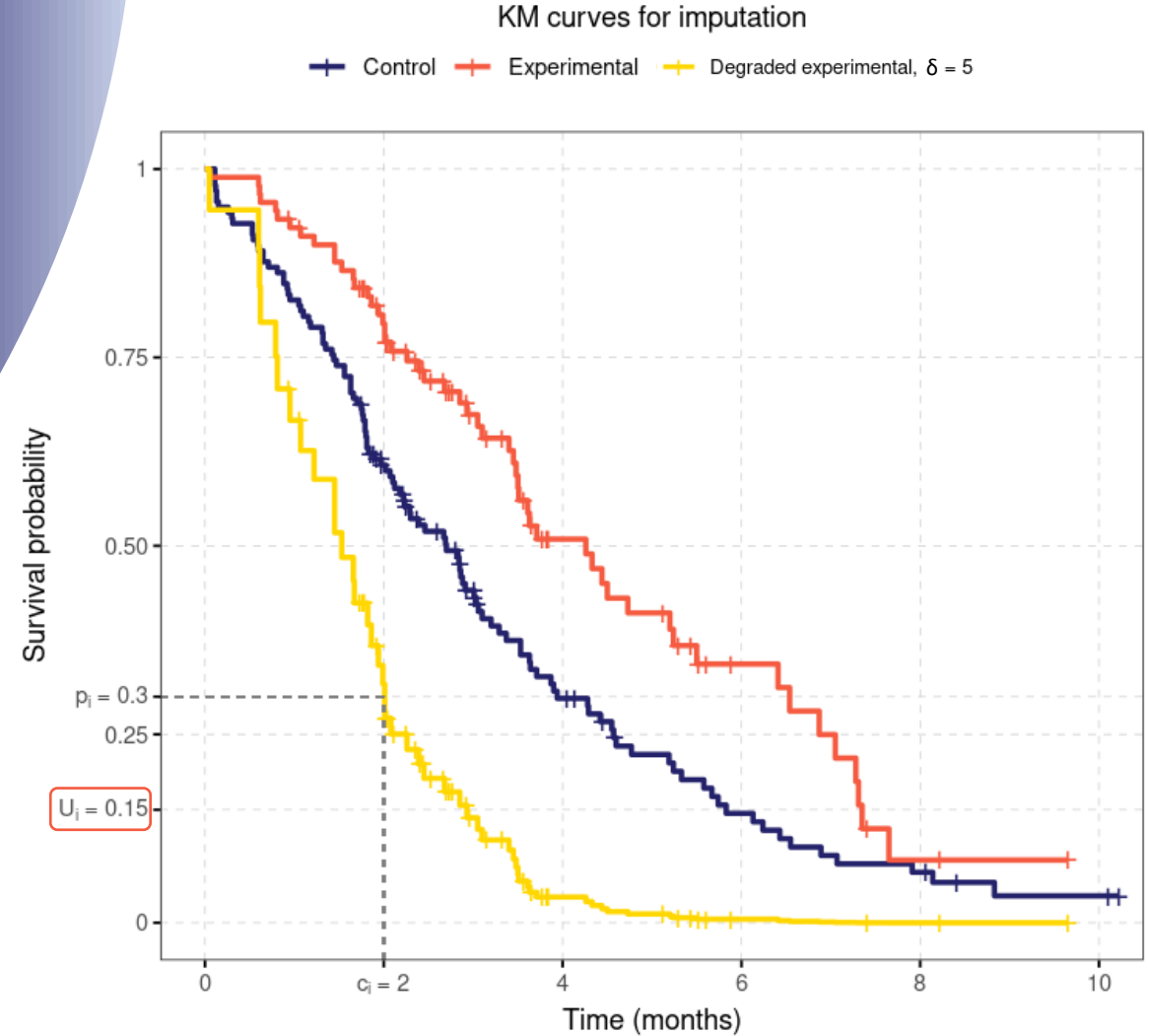
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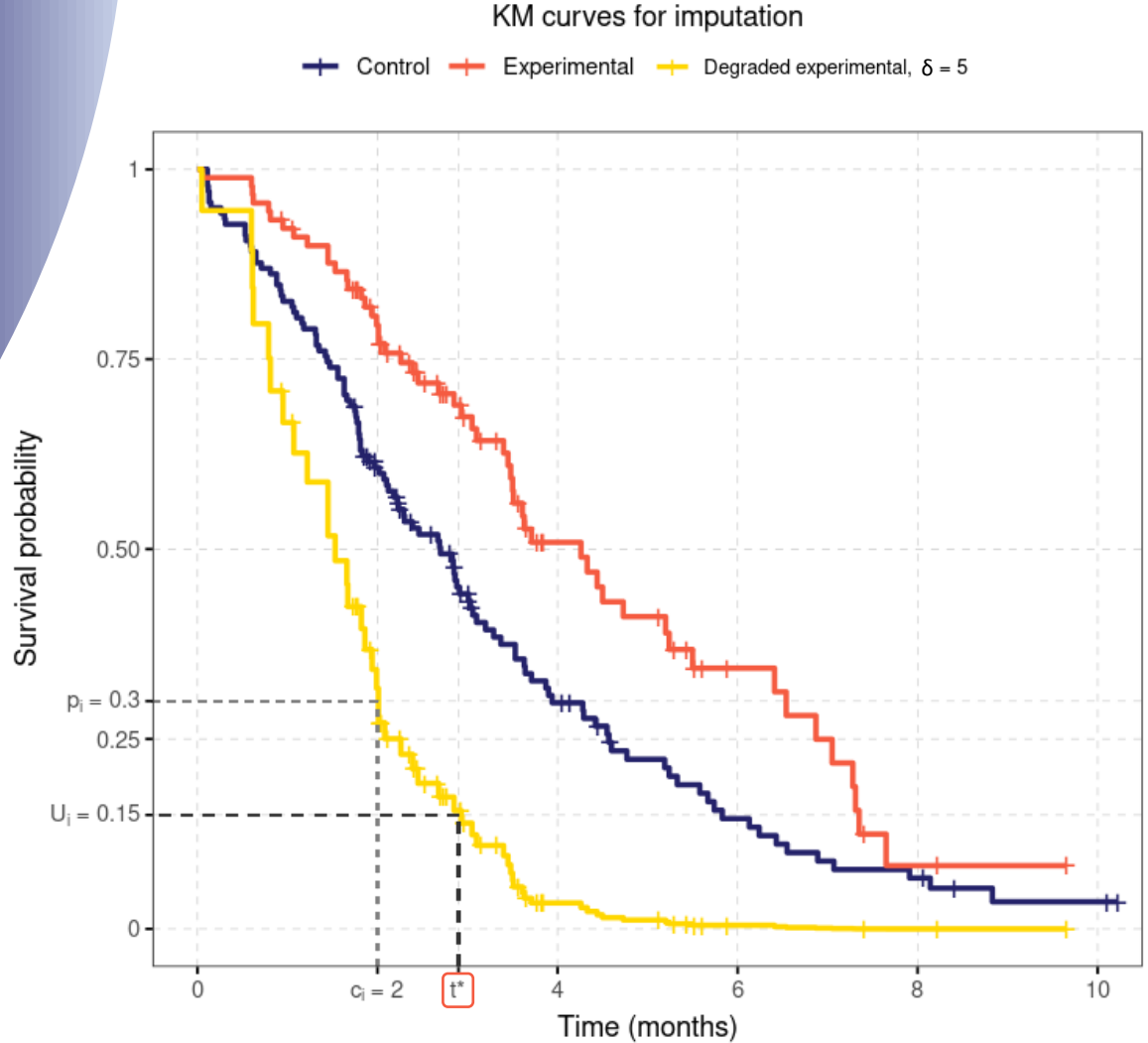
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- Repeat  $m$  times
- Pool results by using Rubin's rules (multiple imputation)

Donald B. Rubin [1987]

# Studied methods

To perform TPA, different methods can be used to estimate  $S(t)$ .  
The other steps of the algorithm remain the same.

We investigated 2 methods to estimate  $S(t)$ :

- Non-parametric **Kaplan-Meier** multiple imputation (KMMI)
  - Allows to stratify on any factor assumed to be related to survival or censoring
- **Cox** proportional hazards multiple imputation (COXMI)
  - Allows to stratify the imputation method to be aligned with the usual model used for survival analysis (Cox model)

# Result process

## Scenario / Simulation

Simulate 2-arm trial dataset

**For non-administratively censored patients, simulate their event times:**

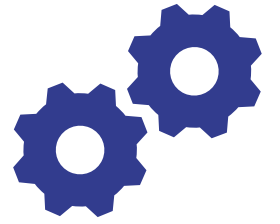
- by **penalizing** their hazard by  $\delta$  and study for which value of  $\delta$  ( $\delta_{\text{theoretical}}$ ) the result switches to non-significant (Tipping Point).

## Evaluation method

**Apply TPA** on the original (non-penalized) censored dataset and observe for **which** value of  $\delta$  it **switches to non-significant** ( $\delta_{\text{imputation}}$ ).

**Compare the theoretical  $\delta$  ( $\delta_{\text{theoretical}}$ ) and the  $\delta$  retrieved by imputation ( $\delta_{\text{imputation}}$ ) using TPA.**

# Simulation setup



## Common parameters

- 2 arms
- Sample size = 800 (1:1)

## Varying parameters

- HR = {0.70; 0.80; 0.85}
- $C_{na}$  = {5%; 10%}
- Accrual time & study duration

For each scenario,

- **1000** datasets are simulated
- **Both** TPA methods (**COXMI** and **KMMI**) are applied



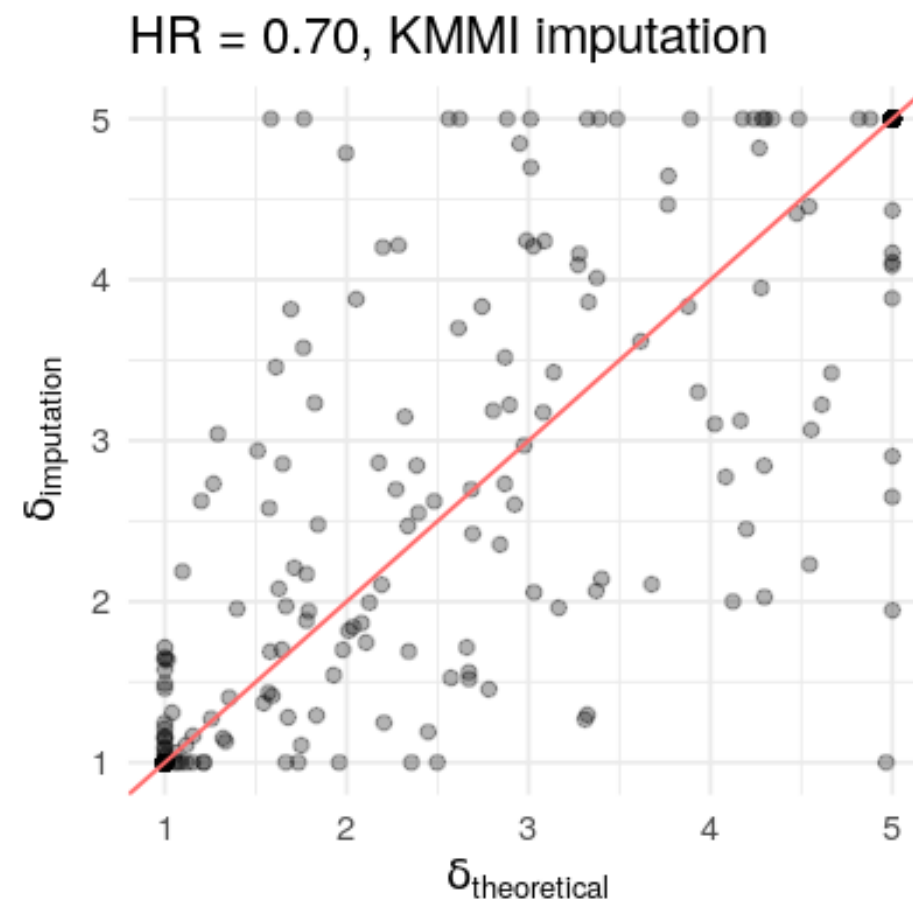
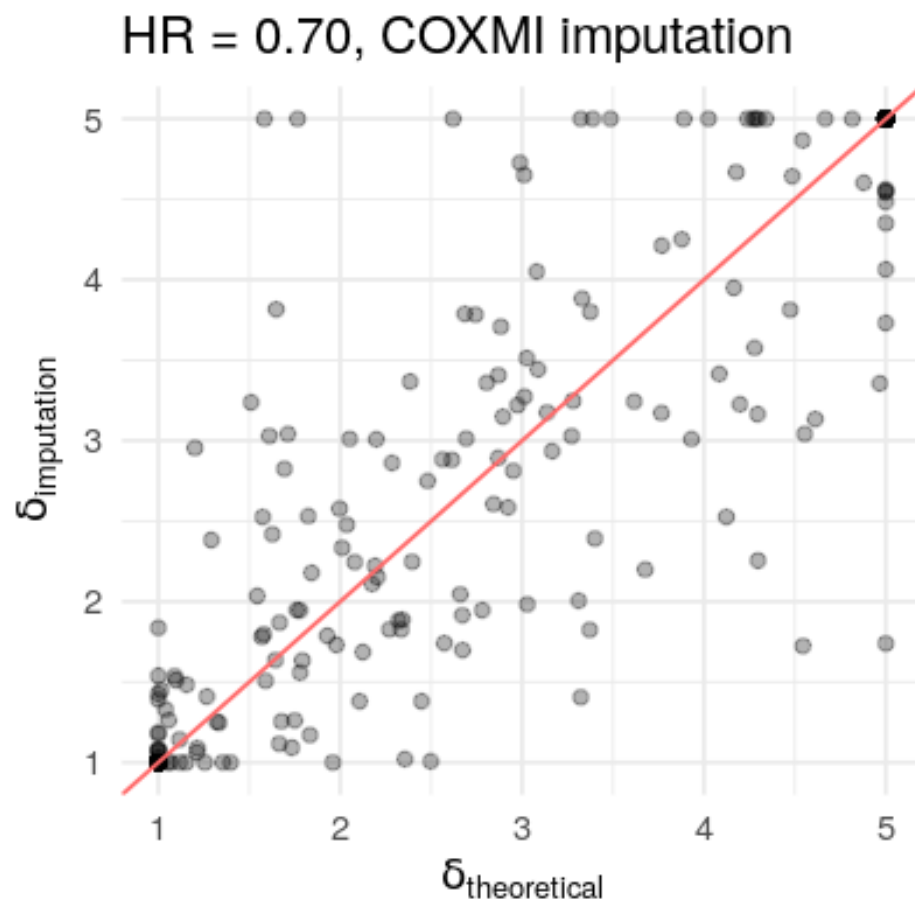
# Results

Observation

	Scenario					
	Sample size = 800 (1:1)					
<b>HR</b>	0.70	0.70	0.80	0.80	0.85	0.85
<b>C<sub>na</sub></b>	5%	10%	5%	10%	5%	10%
Estimated $\delta_{\text{theoretical}}$	5	4.06	5	1.65	1.20	1.10
Estimated $\delta_{\text{imputation}}$ <b>COXMI</b>	5	4.38	5	2.18	1.37	1.12
Estimated $\delta_{\text{imputation}}$ <b>KMMI</b>	5	4.52	5	2.28	1.39	1.12
MSE (COXMI)	0.145	0.271	0.442	0.453	0.425	0.606
MSE (KMMI)	0.381	0.366	0.549	0.550	0.487	0.731

# Results

Observation



# Results

## Interpretation

Based on our simulations:

- In average, TPA based on COXMI/KMMI is efficient for recovering the theoretical  $\delta$  value of a clinical trial
- Choice of the method (COXMI / KMMI) should be motivated by their pros and cons
- The  $\delta$  value might be driven by:
  - sample size
  - informative censoring rate
  - informative censoring times distribution





## Conclusion

- TPA can be used to test the **robustness** of results to deviations from the ignorable censoring assumption (CAR).
- It provides **clinically interpretable** results
- Range of methods allows to **match** a method with the analysis planned for a particular clinical trial
- Tipping Point value is driven by different parameters of a clinical trial

# To go further

## Explore other methods

to test the ignorable censoring assumption

- Copy-reference,
- Jump-to-reference
- ...

## Explore and compare TPA variants

- improving control arm ( $\delta < 1$ ),
- imputing only the experimental arm
- ...

## Produce a user guide

with recommendations on the most appropriate method to use, depending on the studied case.

# References

Ilya Lipkovich, Bohdana Ratitch & Michael O'Kelly. Sensitivity to censored-at-random assumption in the analysis of time-to-event endpoints.  
*Pharmaceutical Statistics; 2016*

Donald B. Rubin. *Multiple Imputation for Nonresponse in Surveys*. John Wiley and Sons Inc. New York, 1987

Thank you for your attention



# Questions ?



# Rubin's rules

Estimate pooling

$$\bar{\theta} = \frac{1}{m} \left( \sum_{i=1}^m \theta_i \right)$$

$\bar{\theta}$  Pooled parameter estimate

$\theta_i$  Parameter estimated at the  $i^{\text{th}}$  imputation

$m$  Number of imputations

Total variance pooling

$$V_{Total} = V_W + V_B + \frac{V_B}{m}$$

$$V_W = \frac{1}{m} \sum_{i=1}^m SE_i^2$$

$$V_B = \frac{\sum_{i=1}^m (\theta_i - \bar{\theta})^2}{m - 1}$$

$V_B$  Between imputation variance

$V_W$  Within imputation variance

Wald testing

$$Wald_{Pooled} = \frac{(\bar{\theta} - \theta_0)^2}{V_{Total}}$$

Pooled wald value follows a t-distribution, used to derive a p-value

$\theta_0$  Parameter value under the null hypothesis

# Simulation scenarios

Scenario	n	HR	C <sub>na</sub>	median <sub>exp</sub>	median <sub>control</sub>	Accrual period	Study duration
1	800 (1:1)	0.7	5%	14.3	10	12	12
2	800 (1:1)	0.7	10%	14.3	10	12	12
3	800 (1:1)	0.80	5%	12.5	10	36	36
4	800 (1:1)	0.80	10%	12.5	10	36	36
5	800 (1:1)	0.85	5%	11.8	10	48	48
6	800 (1:1)	0.85	10%	11.8	10	48	48

n: sample size

C<sub>na</sub>: non-administrative censoring rate

# Tipping point algorithm

